Total No. of Questions : 8] | Total No. of Printed Pages : 4

Paper Code : 41101/45501/45551

B.A./B.Sc. (First Semester)

(Main/Back) Examination, 2025-26

MATHEMATICS

Paper : (B-501)

Differential Calculus & Integral Calculus

Time : Two Hours / *Maximum Marks : 75*

Note : Attempt **all** the sections as per instructions.

Section-A

(Very Short Answer Type Questions)

Note : Attempt all **ten** questions. Each short question carries **2** marks. 10×2=20

1. (a) Evaluate $\int_0^2 \int_0^{\sqrt{4-x^2}} \frac{dx\,dy}{4+x^2+y^2}$

(b) Define upper and lower Riemann integral.

(c) Write the statement of Leibnitz's theorem.

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(d) Find the envelope of the family of straight lines

$y = mx + \frac{1}{m^2}$, the parameter being m.

(e) Show that the sequence $\left\langle \frac{1}{n} \right\rangle$ has the limit 0.

(f) Evaluate :

$$\lim_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\frac{4}{n^2} + \frac{3}{n^2}\sec^2\frac{9}{n^2} + \ldots\ldots + \frac{1}{n}\sec^2 1\right]$$

(g) If $\mu = f\left(\frac{y}{x}\right)$, show that $x\frac{\partial \mu}{\partial x} + y\frac{\partial \mu}{\partial y} = 0$.

(h) If $\vec{r} = \sin t\,\hat{i} + \cos t\,\hat{j} + t\hat{k}$, then find $\left|\frac{d^2\vec{r}}{dt^2}\right|$.

(i) State Stokes theorem.

(j) Evaluate :

$$\lim_{x\to\infty}\left(1 + \frac{a}{x}\right)^x.$$

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Section-B

(Short Answer Type Questions)

Note : Attempt any **two** questions. Each short question carries **15** marks. Short answers are required not exceeding **200** words.

15×2=30

2. Test the convergence of the integral $\int_0^\infty \frac{\sin^2 x}{x^2} dx$.

3. Show that the function f defined on R by

$f(x)=|x|+|x-1|$

is continuous but not differentiable at the points 0 and 1. https://www.mjpruonline.com

4. Prove that :

$\overline{(m)}\left|\left(m+\frac{1}{2}\right)\right. = \frac{\sqrt{\pi}}{2^{2m-1}}\overline{(2m)}$, where $m > 0$.

5. Prove that the radius of curvature at any point t of the cycloid $x=a(t+\sin t)$, $y=a(1-\cos t)$ is given by : $p = 4a\cos\frac{t}{2}$.

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Section-C/ खण्ड-स

(Long Answer Type Questions)

(दीर्घ उत्तरीय प्रश्न)

Note : Attempt any **one** question. Short questions carries 25 marks. Long answer are required not exceeding 800 words.

केवल एक प्रश्न का उत्तर दीजिये। दीर्घ उत्तरीय प्रश्न 25 अंकों का है। अधिकतम 800 शब्दों में दीर्घ उत्तर अपेक्षित हैं।

6. State and prove Lagrange's mean value theorem.

लेग्रांज मध्यमान प्रमेय का कथन लिखकर सिद्ध कीजिये।

7. Prove that every bounded sequence has at least one limit point.

सिद्ध कीजिये कि प्रत्येक बंधित अनुक्रम का कम से कम एक लिमिट बिन्दु होता है।

8. Solve $\iiint \log_e(x+y+z)\,dxdydz$ over a region $x \geq 0$, $y \geq 0$, $z \geq 0$, and $x+y+z \leq 1$.

$x \geq 0$, $y \geq 0$, $z \geq 0$, तथा $x+y+z \leq 1$ द्वारा घिरे हुए क्षेत्र के लिए समाकलन $\iiint \log_e(x+y+z)\,dxdydz$ का मान ज्ञात कीजिए।

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