B Sc (Part-I) First Semester (NEP) Winter 2025

126200 Major Theory-1 - Mathematics - Algebra and Calculus

P Pages 2
Time Two Hours

AI - 4562
Max Marks : 30

Note From each unit solve (a) or (b) and (c) or (d)

UNIT-I

1. a) Find the row rank and column rank of a matrix 4

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 5 \\ 2 & 4 & 8 \end{bmatrix}$$

OR

b) Verify Cayley-Hamilton's theorem for the matrix 4

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$$

c) Show that, the eigenvalues of a Hermitian matrix are all real 4

OR

d) Verify Cayley - Hamilton theorem for the matrix 4

$$M = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Hence find M^4

UNIT-II

2. a) If n is negative integer then prove that 4
$(\cos\theta + i\sin\theta)^n = (\cos n\theta + i\sin n\theta)$

OR

b) If $\sin(A + iB) = x + iy$, then show that 4

i) $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$

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ii) $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$

c) Prove that, $\operatorname{sech}^2 x + \tanh^2 x = 1$ 3

OR

d) Prove that, $\cosh^2 x - \sinh^2 x = 1$ 3

UNIT-III

3. a) Let $\lim_{x \to x_0} f(x) = \ell$ and $\lim_{x \to x_0} g(x) = m$ and these limits exists, then prove that 4

$\lim_{x \to x_0} [f(x) + g(x)] = \ell + m$.

OR

b) Show that $\lim_{x \to 3} x^2 = 9$ using $\epsilon - \delta$ definition of limit 4

c) Prove that the function $f(x) = \sin x$ is continuous for all real values of x 4

OR

d) Show that $f(x) = \begin{cases} \frac{e^{1/x}}{1+e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ 4

has a simple discontinuity at $x = 0$.

UNIT-IV

4. a) State and prove Lagrange's mean value theorem 4

OR

b) State and prove Cauchy's mean value theorem 4

c) Find the value of 'C' for the function which satisfy Rolle's theorem 3

$f(x) = x^2 - 2x - 8 \text{ in } [-1, 3]$

OR

d) Obtain the Maclaurin's series for $f(x) = \cos x$ 3

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